Mathematical Proof (Foundation)

Even numbers

Let m and n be positive integers

a) $2n$ and $2m$ are even numbers.
Why?
Any multiple of 2 is an even number

b) $2n$ and $2m$ are the same even number if $n = m$
Why?
Because we then have the identity $2n \equiv 2m$

c) Show that the product of two distinct even numbers is also even.
If $n \neq m$, $2n$ and $2m$ are distinct.
$2n \times 2m = 4mn$ or 2^2mn which is a multiple of 2,
so is always even

d) Show that the square of an even number is also even.
$(2n)^2 = 4n^2$ or 2^2n^2 which is a multiple of 2,
so is always even

e) Show that the sum of two even numbers is even.
$2n + 2m = 2(m + n)$ which is a multiple of 2,
so is always even

Odd numbers

Let m and n be positive integers

a) As m and n are positive integers, $2m$ and $2n$ are both even $2n - 1$ and $2m - 1$ are odd numbers. Why?
One less than an even number is an odd number

b) Show that the product of two distinct odd numbers is also odd.
If $n \neq m$, $2n - 1$ and $2m - 1$ are distinct.
$(2n - 1)(2m - 1)$
$= 4mn - 2m - 2n + 1$
$= 2(2mn - m - n) + 1$
Since $2(2mn - m - n)$ is even,
$2(2mn - m - n) + 1$ must be odd

c) Show that the square of an odd number is also odd.
$(2n - 1)^2$
$= 4n^2 - 4n + 1$
$= 4(n^2 - n) + 1$

Mathematical arguments

a) Show algebraically that
$4(3x - 5) - 3 \equiv 12x - 23$
Expanding the LHS
$12x - 20 - 3 \equiv 12x - 23$
$12x - 23 \equiv 12x - 23$

b) Show algebraically that
$2(n + 2)(n - 1) - 2(1 - n) \equiv (n - 1)(n + 3)$
Expanding the LHS
$2(n^2 + n - 2) - 2 + n \equiv (n - 1)(n + 3)$
$2n^2 + 2n - 4 - 2 + 2n \equiv (n - 1)(n + 3)$
$2n^2 + 4n - 6 \equiv (n - 1)(n + 3)$
Dividing the LHS by 2
$n^2 + 2n - 3 \equiv (n - 1)(n + 3)$
Factorising the LHS
$(n - 1)(n + 3) \equiv (n - 1)(n + 3)$

c) Show algebraically that
$(n + 7)^2 - (n + 1)^2 \equiv 12(n + 4)$
Expanding the LHS
$n^2 + 14n + 49 - n^2 - 2n - 1 \equiv 12(n + 4)$
Simplifying the LHS
$12n + 48 \equiv 12(n + 4)$
Factorising the LHS
$12(n + 4) \equiv 12(n + 4)$

Consecutive numbers

n, $n + 1$ and $n + 2$ are consecutive numbers.
How would you write three consecutive even numbers?

$2n$, $2n + 2$, $2n + 4$

Giving counterexamples

a) The sequence generated by $4n - 1$ contains infinitely many primes, but not every number in this sequence is prime. Give an example of a number from this sequence that is not prime.

Possibilities include 15, 27, 35, etc

b) Shaq thinks that $(3n)^3$ is always positive if n is an integer. Find a counterexample to show that Shaq is incorrect.

Any case where $n < 0$